Random Walk in Complex Systems with the Particle Diffusion Model - PDM

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Communication networks in real systems in several cases have limited resources. The bandwidth between the nodes (computers, cell phones or other wireless devices) allows for the transmittance of data with specific rates. The data transmitted can, and in most cases have, different importance for the user. Some data are more useful than other. An example can be a peer to peer network, like skype, were video and audio data are transmitted along with text. The text can be considered as more important than both audio and video, with audio being more important than video. In a limited bandwidth situation (network overload or broken main connection) we may be required to at least have the ability to write. This means that priority must be given to the transmission of text.

We use a model for the motion of two types of particles, *A* and *B*, with priority, which can represent any type of information [1]. We investigate several protocols for the diffusion of particles, based on either a random choice of site or that of a randomly chosen particle out of both species. We define them respectively as "site" and "particle" protocols. In all cases, when both species coexist in the same space, the movement of *A* takes precedence over that of *B*. Assigning an opposite priority would simply produce symmetric results. We apply strict priority of *A* over *B*, meaning that *B* will not move until all coexisting *A* have left the same site. To find the probability of a *B* leaving the site we must therefore find the probability of a site being empty of *A*, f_0.

We initiate our study from lattices and then move on to more complex systems like networks (random graphs, scale-free, fractal and real world networks). Analytic and simulation results confirm that the site protocol depends inversely on the density of *A*, ρ_A, while the particle protocol exhibits an inverse exponential behavior. This means that when the particle protocol is applied in lattices, the system tends to go into an equilibrium state much faster than with the site protocol and the mobility of *B* is lower. Results show that both species execute normal diffusion, although the *B* move slower than the *A*. We calculate the normalized diffusion coefficient Φ of these systems and find the dependence on density (Figure 1).

In networks, for the particle protocol, it is found that the number of sites empty of the high priority population depends exponentially on the degree, *k*, (Figure 2). As previously in lattices the motion of either species is also strongly related to the density of *A*. It is interesting though to mention that this dependence on either parameter is the same in various kinds of networks with totally different topologies like random graphs and scale-free. The movement of *B* decreases exponentially with the degree, while for *A* it increases. This is due to two reasons: a) the higher the degree, the more likely that a *B* will lose its movement, thus giving it to an *A* and b) the higher the degree, more *A* will move through this node and particles from that node will be chosen more frequently. Therefore, the installment of priorities leads to a faster movement of *A* and the concentration of *B* at the hubs, where their motion is hindered. This can be easily seen in the waiting time distribution for *B* particles of a specific degree (Figure 3). It is obvious that for low node degree the waiting time of *B* is short, while for high degree nodes it is significantly increased. Simulation results from the average waiting time, $\langle t \rangle$, versus the degree (Figure 4) present a similar picture. Also verified is the existence of finite size (networks of finite size) and time (finite number of Monte Carlo Steps) effects, as we had expected. Additionally, the average number of particles per node is exponential to the degree. For regular real world networks the coverage of small degree networks is very low and they are essentially empty of both species most of the time. The high degree nodes are rich in particles and the low degree are poor.

In conclusion, we have presented a novel way of information transmission through random walk with priorities in complex systems. This priority diffusion model can have important technological applications in communication networks as well as possibly other social, ecological or even roadway traffic systems. In lattices and regular networks the two species move as two non interacting ones with different mobility. In scale free (and real world) networks the low priority species is mired in the central hubs while the high priority ones move faster. This effect can be utilized to selectively halt all diffusion of certain types of information in a network and dedicate all available bandwidth in the high priority information.

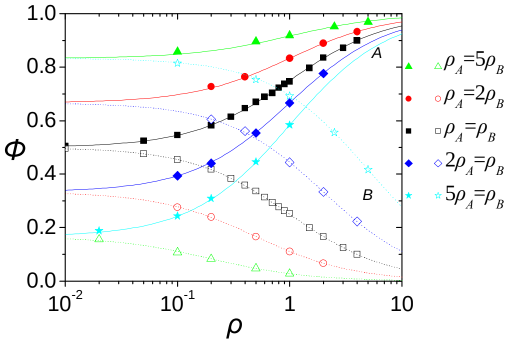

Figure 1. Diffusion coefficient over the density for various ratios of densities. Lines are the corresponding theory. Full lines and symbols represent *A* and hollow symbols and dash lines *B* particles.

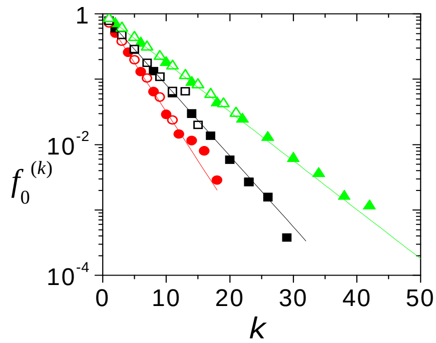

Figure 2. The number of empty sites per node degree. Full symbols are results from scale-free networks, hollow from ER. The lines are the corresponding theory.

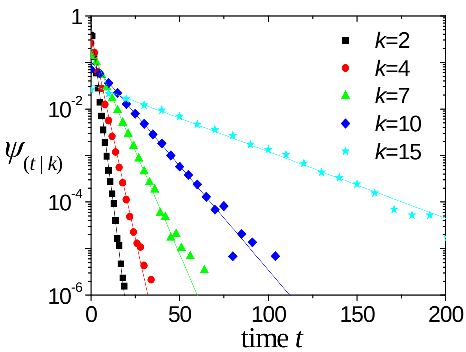

Figure 3. The distribution of waiting times of *B* particles for various node degrees. The higher the degree the higher the probability of a *B* particle to stay more time in that node. Lines are theoretical results, symbols are simulations. For higher order degrees the results become independent of time (not shown due to poor statistics).

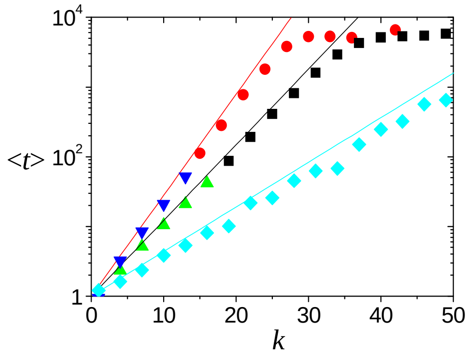

Figure 4. Average waiting time of *B* particles over the node degree. For high values of $\langle t \rangle$ there are finite time effects. Blue and green triangles are ER results, black and red are for scale free. The cyan are results from a real computer network (AS level internet) where PDM is applied. Lines are corresponding theory.

[1] M. Maragakis, S. Carmi, D. ben Avraham, S. Havlin, and P. Argyrakis, Phys. Rev. E Rapid Communication, 77, 020103 (2008).

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